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POLYNOMIAL SYSTEMS WITH ENOUGH INVARIANT ALGEBRAIC CURVES

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Abstract. Assuming that a planar system of degree n has $(n-1)(n+2)/2$ invariant algebraic curves (taking into account their multiplicities) we prove that this system has a finite number of limit cycles. Furthermore this number only depends on the fundamental groups of the connected components of the points of the plane which are not on these algebraic curves

INTRODUCTION AND STATEMENT OF MAIN RESULTS

A planar polynomial real differential equation

$$\dot{x} = P(x,y) \quad , \quad \dot{y} = Q(x,y), \tag{1}$$

where P and Q are polynomials of degree at most n and at least one of them has this degree will be called a polynomial system of degree n. We will say that $X = (P,Q)$ is a PSn.

An algebraic curve $R(x,y) = 0$ is a solution of (1) if $R(x,y) \in \mathbb{R}[x,y]$ and $< \nabla R(x,y), X(x,y) > |_{R(x,y)=0} = 0$ (on $\mathbb{C}^2$). If we assume that R is product of irreducible factors we have that if $R = 0$ is an algebraic solution of (1) then

$$R(x,y) \ \ | < \nabla R(x,y), X(x,y) >,$$

see [G]. Hence we will give the following definition:

$R(x,y) = 0$ is an *algebraic solution* of (1) *with multiplicity* $m \in \mathbb{N}$ if $R \in \mathbb{R}[x,y]$,

$$R^m(x,y) \ | < \nabla R(x,y), X(x,y) > \text{ and } R^{m+1}(x,y) \nmid < \nabla R(x,y), X(x,y) > .$$

The polynomials $\frac{<\nabla R, X>}{R}, \frac{<\nabla R, X>}{R^2}, \ldots, \frac{<\nabla R, X>}{R^m}$ are called the *quotients* of the algebraic solution $R = 0$. Note that an algebraic solution of multiplicity m has associated m quotients.

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