DIFFERENTIAL EQUATIONS THAT CAN BE TRANSFORMED INTO EQUATIONS OF LIENARD TYPE

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Abstract. The aim of this paper is to show how the study of several kind of planar differential equations can be reduced to the study of equations of Liénard type.

1. INTRODUCTION

E. and C. Cartan [CC] in a radiothechnical question and B. Van der Pol [V1],[V2] in a problem of electricity considered particular cases of the so called Liénard equations, see [L]:

$$\ddot{x} + f(x)\dot{x} + g(x) = 0. \tag{1}$$

This equation, taking coordinates $w = \dot{x}$ and x is equivalent to system

$$\begin{cases} \dot{w} = -g(x) - f(x)w, \\ \dot{x} = w, \end{cases} \tag{2}$$

and in coordinates x, and $y = \dot{x} + F(x)$, where $F(x) = \int_0^x f(u)\, du$, it becomes

$$\begin{cases} \dot{x} = y - F(x), \\ \dot{y} = -g(x). \end{cases} \tag{3}$$

Phase portraits of equations (2) and (3) have been studied for a long time. There are many criteria to ensure non existence or uniqueness of limit cycles for such equations. Excellent surveys are the book of Sansone and Conti [SC], [C2] or [S]. Other approaches can be found in [Co2] or [CGL]. There are also several results about Liénard equations with more than one limit cycle, see for instance [Ll], [R] or [Su].

In this paper we consider several types of planar differential system for which we can show that the study of their number of limit cycles can be reduced to the study of a differential equation of Liénard type. Most important types are quadratic systems and predator-prey systems.

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