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Limit Cycles of Polynomially Integrable Piecewise Differential Systems

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Abstract: In this paper, we study how many algebraic limit cycles have the discontinuous piecewise linear differential systems separated by a straight line, with polynomial first integrals on both sides. We assume that at least one of the systems is Hamiltonian. Under this assumption, piecewise differential systems have no more than one limit cycle. This study characterizes linear differential systems with polynomial first integrals.

Keywords: algebraic limit cycle; piecewise differential system; polynomial integrable systems

MSC: 34C05; 34C07; 37G15

1. Introduction

We will consider discontinuous piecewise linear differential systems (DPwLS) on the plane $\mathbb{R}^2$. On the half plane where x is negative they are expressed as:

$$\begin{pmatrix} \dot{x} \\ \dot{y} \end{pmatrix} = A^- \begin{pmatrix} x \\ y \end{pmatrix} + b^-, \tag{1}$$

and where x is positive as:

$$\begin{pmatrix} \dot{x} \\ \dot{y} \end{pmatrix} = A^+ \begin{pmatrix} x \\ y \end{pmatrix} + b^+, \tag{2}$$

where $A^{\pm}$ are real 2×2 matrices and $b^{\pm} \in \mathbb{R}^2$. For the definition of a DPwLS on $x = 0$, we follow Filippov's solution [1].

DPwLSs have been studied in depth [2]. An introduction to and a comprehensive list of references can be found in the books [3,4], and the survey [5].

Since planar linear differential systems have no limit cycles (isolated periodic orbits), the limit cycles of DPwLSs separated by a straight line must cross the straight line at two points. In this paper, we do not consider the possible limit cycles which have a segment on the discontinuous straight line, called sliding limit cycles.

The limit cycles of planar differential systems play a main role in understanding the dynamics of such systems, as well as for planar DPwLSs. Thus, the limit cycles of DPwLSs (1) and (2) have been studied intensively in the last twenty years. The current situation of proven bounds is summarized in [6].

One problem still to be solved is: "*Is three the maximum number of limit cycles that a discontinuous piecewise linear differential system with a straight line of separation can have*?"

Recently, Buzzi, Gasull and Torregrosa analyzed the particular class of algebraic limit cycles in the DPwLS (1) and (2) [7]. They establish that a limit cycle is *algebraic* if "all its points, except the ones on the sliding set, are contained in the level sets of one or two

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