Mediterr. J. Math. (2021) 18:221
https://doi.org/10.1007/s00009-021-01845-y
1660-5446/21/050001-27
published online September 7, 2021

Mediterranean Journal of Mathematics

Topological Properties of the Immediate Basins of Attraction for the Secant Method

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Abstract. We study the discrete dynamical system defined on a subset of R^2 given by the iterates of the secant method applied to a real polynomial p. Each simple real root α of p has associated its basin of attraction $\mathcal{A}(\alpha)$ formed by the set of points converging towards the fixed point (α, α) of S. We denote by $\mathcal{A}^*(\alpha)$ its immediate basin of attraction, that is, the connected component of $\mathcal{A}(\alpha)$ which contains (α, α). We focus on some topological properties of $\mathcal{A}^*(\alpha)$, when α is an internal real root of p. More precisely, we show the existence of a 4-cycle in $\partial\mathcal{A}^*(\alpha)$ and we give conditions on p to guarantee the simple connectivity of $\mathcal{A}^*(\alpha)$.

Mathematics Subject Classification. 37G35, 37N30, 37C70.

Keywords. Root finding algorithms, rational iteration, secant method, periodic orbits.

1. Introduction and Statement of the Results

Dynamical systems is a powerful tool to have a deep understanding on the global behavior of the so called *root-finding* algorithms, that is, iterative methods capable to numerically determine the solutions of the equation $f(x) = 0$. In most cases, it is well known the order of convergence of those methods near the zeros of f, but it is in general unclear the behavior and effectiveness when initial conditions are chosen on the whole space; a natural question when we do not know a priori where the roots are or if there are many of them.

The numerical exploration of the solutions of the equation $f(x) = 0$ has been always central problem in many areas of applied mathematics, from biology to engineering, since most mathematical models require to have a thorough knowledge of the solutions of certain equations. Once we are certain that no algebraic manipulation of the equation will allow to explicitly find

This work has been partially supported by MINECO-AEI grants MTM-2017-86795-C3-2-P and MTM-2017-86795-C3-3-P and the AGAUR grant 2017 SGR 1374.