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Global Dynamics of the Hořava–Lifshitz Cosmological Model in a Non-Flat Universe with Non-Zero Cosmological Constant

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Abstract: When the cosmological constant is non-zero, the dynamics of the cosmological model based on Hořava–Lifshitz gravity in a non-flat universe are characterized by using the qualitative theory of differential equations.

Keywords: global dynamics; Hořava–Lifshitz; non-flat universe; cosmology constant

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1. Introduction

After the Newtonian era, Einstein put forward the general theory of relativity, and this gave our understanding of gravity, once again, a huge leap forward. In 2009 Hořava proposed a non-relativistic theory of renormalizable gravity [1], which can be reduced to Einstein's general theory of relativity on a large scale. It is named Hořava–Lifshitz gravity together with the scalar field theory of Lifshitz. This theory has inspired many studies and applications in length renormalization [2], entropy argument [3], cosmology [4–30], dark energy [31–35], black holes [36–40], gravitational waves [41] and electromagnetics [42–45]. More information can also be found in the review articles [46–48] and the references therein.

In the past ten years, Leon et al. [9–12] have conducted several excellent studies on the Hořava–Lifshitz cosmological model, whether the curvature k of the universe is zero and whether the cosmological constant Λ should be considered. They divided the cosmological model into four types in the Friedmann–Lemaître–Robertson+-Walker (FLRW) background spacetime: (1) $\Lambda = 0, k = 0$; (2) $\Lambda \neq 0, k = 0$; (3) $\Lambda = 0, k \neq 0$; (4) $\Lambda \neq 0, k \neq 0$. By using the phase-space analysis, they discussed the two-dimensional dynamics of the Hořava–Lifshitz cosmological model under the usual exponential potential, and partially studied its three-dimensional dynamics.

For the important cosmological constant Λ that many researchers have been paying attention to, this constant put the Hořava–Lifshitz gravitational theory in a delicate balance, which led to a conflict between its cosmology and observations. Appignani et al. [49] showed that the huge difference between the standard predictions from quantum field theory and the observed value of Λ may have a solution in the Hořava–Lifshitz gravity framework. Akarsu et al. [50] investigated the Λ in the standard cold dark matter model by introducing graduated dark energy. Their results provided a high probability that the sign of Λ could be spontaneously converted, and inferred that the universe had transformed from anti-de Sitter vacua to de Sitter vacua and triggered late acceleration. Carlip [51] proposed that the vacuum fluctuations produce a huge Λ and produce a high curvature k on the Planck scale under the standard effective field theory. Although the debate about the shape of the universe has not yet reached an agreement, for the boundaries of the universe are blurred, the odds are 50:1 that the universe is closed if the Planck CMB data are correct [52]. Besides, Valentino et al. [53] also believed that the universe could be a closed three-dimensional sphere, unlike the prediction from the standard Λ cold dark matter