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Note

A Note on the Periods of Surface Homeomorphisms

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If f is a surface homeomorphism isotopic to a pseudo-Anosov one, then its set of periods is cofinite, i.e., there exists a finite subset S of $\mathbb{N}$ such that the set of periods of f is equal to $\mathbb{N} \setminus S$. © 1993 Academic Press, Inc.

1. INTRODUCTION

Let M be a compact connected oriented surface possibly with boundary. A homeomorphism $F: M \to M$ is said to be *pseudo-Anosov* if there is a real number $\lambda = \lambda(F) > 1$ and a pair of transverse measured foliations $\mathbf{F}^s$ and $\mathbf{F}^u$ such that $F(\mathbf{F}^s) = \lambda^{-1}\mathbf{F}^s$ and $F(\mathbf{F}^u) = \lambda\mathbf{F}^u$. Pseudo-Anosov homeomorphisms are topologically transitive, have positive entropy, and have Markov partitions [5]. More concretely, let $\{R_1, R_2, ..., R_k\}$ be a Markov partition for F. It is known that F is semi-conjugate to the subshift of finite type (Σ_k, A, σ) defined as follows. Set $\Sigma_k = \{1, 2, ..., k\}^{\mathbb{Z}}$, $A = A(F)$ the $k \times k$ transition matrix defined by $a_{ij} = 1$ if $F(\operatorname{Int} R_i) \cap \operatorname{Int} R_j \neq \varnothing$, and $a_{ij} = 0$ otherwise, and $\sigma: \Sigma_k \to \Sigma_k$ the shift map. If $\{b_i\} \in \Sigma_k$ then $\bigcap_{i \in \mathbb{Z}} F^{-1}(R_{b_i})$ consists of a single point, and the semi-conjugacy $h: \Sigma_k \to M$, satisfying $h \circ \sigma = F \circ h$, is given by $h(\{b_i\}) = \bigcap_{i \in \mathbb{Z}} F^{-1}(R_{b_i})$. Moreover, if $\rho(A)$ is the spectral radius of A, then λ is the unique

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