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Periods of Surface Homeomorphisms

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ABSTRACT. The goal of this paper is to investigate which sets of positive integers can occur as the periods of the periodic orbits of a surface homeomorphism on a given compact surface. We also investigate the influence of the induced map on homology on the sets of periods which can occur.

1. Introduction and statement of results

Compact connected 2-dimensional manifolds are called *surfaces*. Any orientable surface without boundary is homeomorphic to the sphere S^2 or to the torus T^2 or to the connected sum of n tori with $n \geq 2$ (i.e. the n-holed torus). The *genus* of an orientable surface without boundary is the number of torus summands.

Let f be a surface homeomorphism. We denote by $\mathrm{Per}(f)$ the set of periods of all periodic points of f.

Fuller, in **[Fu]**, proved the following result; see also Halpern **[Hl]** and Brown **[Br]**.

THEOREM 1. *Let f be a homeomorphism of a compact polyhedron X into itself. If the Euler characteristic of M is not zero, then f has a periodic point with period not greater than the maximum of $\sum_{k\ odd} B_k(X)$ and $\sum_{k\ even} B_k(X)$, where $B_k(X)$ denotes the k-th Betti number of X.*

If we apply Theorem 1 to surface homeomorphisms we obtain

COROLLARY 2. *Let S be an orientable surface without boundary of genus g and let $f : S \to S$ be a homeomorphism. Then the following statements hold*:

(1) *If* $g = 0$ *then* $\mathrm{Per}(f) \cap \{1, 2\} \neq \emptyset$.
(2) *If* $g > 1$ *then* $\mathrm{Per}(f) \cap \{1, 2, \dots, 2g\} \neq \emptyset$.

PROOF. It is well known that $B_0(S) = B_2(S) = 1$ and that $B_1(S) = 2g$

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