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Stable Components in the Parameter Plane of Transcendental Functions Of Finite Type

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Abstract

We study the parameter planes of certain one-dimensional, dynamically-defined slices of holomorphic families of entire and meromorphic transcendental maps of finite type. Our planes are defined by constraining the orbits of all but one of the singular values, and leaving free one asymptotic value. We study the structure of the regions of parameters, which we call *shell components*, for which the free asymptotic value tends to an attracting cycle of non-constant multiplier. The exponential and the tangent families are examples that have been studied in detail, and the hyperbolic components in those parameter planes are shell components. Our results apply to slices of both entire and meromorphic maps. We prove that shell components are simply connected, have a locally connected boundary and have no center, i.e., no parameter value for which the cycle is superattracting. Instead, there is a unique parameter in the boundary, the *virtual center*, which plays the same role. For entire slices, the virtual center is always at infinity, while for meromorphic ones it maybe finite or infinite. In the dynamical plane we prove, among other results, that the basins of attraction which contain only one asymptotic value and no critical points are simply connected. Our dynamical plane results apply without the restriction of finite type.

Keywords Holomorphic dynamics · Transcendental functions · Parameter spaces

Mathematics Subject Classification 37F10 · 37F46 · 30D30

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