



# The Extended 16th Hilbert Problem for Discontinuous Piecewise Linear Centers Separated by a Nonregular Line

Marina Esteban

*Departamento de Matemática Aplicada II,  
Escuela Técnica Superior de Ingeniería de la Universidad de Sevilla,  
Camino de los Descubrimientos s/n, Sevilla 41092, Spain  
marinaep@us.es*

Jaume Llibre

*Departament de Matemàtiques,  
Universitat Autònoma de Barcelona, 08193 Bellaterra,  
Barcelona, Catalonia, Spain  
jllibre@mat.uab.cat*

Claudia Valls

*Centro de Análise Matemática,  
Geometria e Sistemas Dinâmicos, Departamento de Matemática,  
Instituto Superior Técnico, Universidade de Lisboa,  
1049-001 Lisboa, Portugal  
cvalls@math.tecnico.ulisboa.pt*

Received December 22, 2020; Revised June 2, 2021

The study of the piecewise linear differential systems goes back to Andronov, Vitt and Khaikin in 1920's, and nowadays such systems still continue to receive the attention of many researchers mainly due to their applications. We study the discontinuous piecewise differential systems formed by two linear centers separated by a nonregular straight line. We provide upper bounds for the maximum number of limit cycles that these discontinuous piecewise differential systems can exhibit and we show that these upper bounds are reached. Hence, we solve the extended 16th Hilbert problem for this class of piecewise differential systems.

*Keywords:* Discontinuous piecewise linear system; linear center; nonregular line.

## 1. Introduction and Statement of the Main Result

One of the main interesting objects in the study of differential systems are limit cycles. A *limit cycle* is a periodic orbit of the differential system isolated in the set of all periodic orbits of the system.

Limit cycles have played and are still playing an important role for explaining physical phenomena, see for instance, the limit cycle of van der Pol

equation [van der Pol, 1920, 1926], or the one of the Belousov–Zhabotinsky model [Belousov, 1959; Zhabotinsky, 1964], etc.

The *extended 16th Hilbert problem*, i.e. to find an upper bound for the maximum number of limit cycles that a given class of differential systems can exhibit, is in general an unsolved problem. Only for very few classes of differential system has this problem been solved. For the class of discontinuous