The 16th Hilbert problem for discontinuous piecewise isochronous centers of degree one or two separated by a straight line

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M. Esteban,[1,a)] J. Llibre,[2,b)] and C. Valls[3,c)]

AFFILIATIONS

[1] Dept. Matemática Aplicada II and Instituto de Matemáticas (IMUS), Escuela Técnica Superior de Ingeniería de la Universidad de Sevilla, Camino de los Descubrimientos s/n, 41092 Sevilla, Spain
[2] Dept. Matemàtiques, Universitat Autònoma de Barcelona, 08193 Bellaterra, Barcelona, Catalonia, Spain
[3] Dept. Matemática, Instituto Superior Técnico, Universidade Técnica de Lisboa, Av. Rovisco Pais, 1049–001 Lisboa, Portugal

[a)] **Author to whom correspondence should be addressed:** marinaep@us.es
[b)] jllibre@mat.uab.cat
[c)] cvalls@math.ist.utl.pt

ABSTRACT

In this paper, we deal with discontinuous piecewise differential systems formed by two differential systems separated by a straight line when these two differential systems are linear centers (which always are isochronous) or quadratic isochronous centers. It is known that there is a unique family of linear isochronous centers and four families of quadratic isochronous centers. Combining these five types of isochronous centers, we obtain 15 classes of discontinuous piecewise differential systems. We provide upper bounds for the maximum number of limit cycles that these fifteen classes of discontinuous piecewise differential systems can exhibit, so we have solved the 16th Hilbert problem for such differential systems. Moreover, in seven of the classes of these discontinuous piecewise differential systems, the obtained upper bound on the maximum number of limit cycles is reached.

To solve the 16th Hilbert problem, i.e., to find an upper bound for the maximum number of limit cycles that a given class of differential systems can exhibit, is in general an unsolved problem. For the classes of discontinuous piecewise differential systems here studied, we can obtain the solution using the first integrals of the linear and quadratic isochronous centers.

I. INTRODUCTION AND MAIN RESULTS

We consider planar differential systems of the form

$$\frac{dx}{dt} = P(x,y), \quad \frac{dy}{dt} = Q(x,y),$$

where $P(x,y)$ and $Q(x,y)$ are polynomial functions, and the degree of the systems is the maximum degree of such polynomials. In particular, in this paper we consider discontinuous piecewise differential systems of the form

$$(\dot{x},\dot{y}) = \mathbf{F}(x,y) = \begin{cases} \mathbf{F}^-(x,y) = (f^-(x,y), g^-(x,y)) & \text{if } x < 0, \\ \mathbf{F}^+(x,y) = (f^+(x,y), g^+(x,y)) & \text{if } x > 0, \end{cases} \quad (1)$$

being bi-valued on the separation line $x = 0$. Following Ref. 9, a point $(0,y)$ is a *crossing point* if $f^-(0,y)f^+(0,y) > 0$. If there exists a periodic orbit of the discontinuous differential system (1) having exactly two crossing points, then we call it a *crossing periodic orbit.* A *crossing limit cycle* is an isolated periodic orbit in the set of all crossing periodic orbits of system (1). In what follows for simplicity, we shall say limit cycle instead of crossing limit cycle.

The analysis of planar continuous piecewise linear systems is well established when the number of linear zones is small, see Ref. 33 and the references therein. They frequently appear in many non-linear engineering devices, which are accurately modelled by