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Periodic orbits bifurcating from a Hopf equilibrium of 2-dimensional polynomial Kolmogorov systems of arbitrary degree

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ABSTRACT

A Hopf equilibrium of a differential system in $\mathbb{R}^2$ is an equilibrium point whose linear part has eigenvalues $\pm\omega i$ with $\omega \neq 0$, where $i = \sqrt{-1}$. We provide necessary and sufficient conditions for the existence of a limit cycle bifurcating from a Hopf equilibrium of 2–dimensional polynomial Kolmogorov systems of arbitrary degree. We provide an estimation of the bifurcating small limit cycle and also characterize the stability of this limit cycle.

1. Introduction and statements of the main results

The Hopf bifurcation in some particular classes of Kolmogorov polynomial differential systems in dimension three has been studied in [10,17]. In dimension two there is only a partial study in [3,21]. The objective of this paper is to characterize the generic Hopf bifurcation of the Kolmogorov polynomial differential systems in dimension two. More precisely, we provide sufficient conditions for the existence of a Hopf bifurcation, we estimate the size of the periodic orbit which bifurcates and we control its kind of stability or inestability.

A polynomial differential system

$$\dot{x} = \frac{dx}{dt} = P(x, y), \qquad \dot{y} = \frac{dy}{dt} = Q(x, y),$$

in $\mathbb{R}^2$ has degree n if the maximum of the degrees of the polynomials P and Q is n. A quadratic polynomial vector field $X = (P, Q)$ with x a factor of P and y a factor of Q is a *Lotka–Volterra system*. While an n–degree polynomial vector field $X = (P, Q)$ with x a factor of P and y a factor of Q is a *Kolmogorov system*.

Lotka–Volterra systems were initially considered independently by Lotka in 1925 [19] and by Volterra in 1926 [25], as a model for studying the interactions between two species. Later on Kolmogorov [14] in 1936 extended these systems to arbitrary dimension and arbitrary degree, these kinds of systems are now called Kolmogorov systems.

Many natural phenomena can be modeled by the Kolmogorov systems such as the time evolution of conflicting species in biology [20], chemical reactions [13], hydrodynamics [7], economics [23], the coupling of waves in laser physics [15], the evolution of electrons, ions and neutral species in plasma physics [16], integrability [2], etc.

Here we study the polynomial Kolmogorov systems in the plane, i.e. differential systems of the form

$$\dot{x} = xf(x, y), \quad \dot{y} = yg(x, y), \tag{1}$$

where f and g are polynomials of degree larger than 1. In fact we are interested in the existence of limit cycles of Kolmogorov systems living in the positive quadrant of the plane, and consequently surrounding some equilibrium points (see for instance Theorem 1.31 of [11]) which are in the positive quadrant.

We recall that a *limit cycle* of the Kolmogorov system (1) is a periodic solution of system (1) isolated in the set of all periodic solutions of (1). In general to detect the existence of limit cycles is a difficult problem.

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