Commun Nonlinear Sci Numer Simulat 95 (2021) 105621

Contents lists available at ScienceDirect

Commun Nonlinear Sci Numer Simulat

journal homepage: www.elsevier.com/locate/cnsns

Research paper

The zero-Hopf bifurcations in the Kolmogorov systems of degree 3 in $\mathbb{R}^3$

Érika Diz-Pita[a,*], Jaume Llibre[b], M. Victoria Otero-Espinar[a], Claudia Valls[c]

[a] Departamento de Estatística, Análise Matemática e Optimización, Universidade de Santiago de Compostela, Santiago de Compostela 15782, Spain
[b] Departament de Matemàtiques, Universitat Autònoma de Barcelona, Bellaterra, Barcelona 08193, Spain
[c] Center for Mathematical Analysis, Geometry and Dynamical Systems, Departamento de Matemática, Instituto Superior Técnico, Universidade de Lisboa, Lisboa, 1049-001, Portugal

ARTICLE INFO

Article history:
Received 16 June 2020
Revised 2 November 2020
Accepted 11 November 2020
Available online 19 November 2020

Keywords:
Lotka–Volterra system
Kolmogorov systems
Zero-Hopf bifurcation
Limit cycle

ABSTRACT

In this work we study the periodic orbits which bifurcate from all zero-Hopf bifurcations that an arbitrary Kolmogorov system of degree 3 in $\mathbb{R}^3$ can exhibit. The main tool used is the averaging theory.

© 2020 Elsevier B.V. All rights reserved.

1. Introduction and statement of the main results

Lotka-Volterra systems were initially proposed, independently, by Lotka in 1925 [1] and Volterra in 1926 [2], both in the context of competing species. These Lotka-Volterra systems are differential systems of the form

$$\dot{x} = xP(x,y), \qquad \dot{y} = yQ(x,y),$$

where P and Q are polynomials of degree 1. Later on the *Lotka-Volterra systems* were generalized and considered on arbitrary dimension $n \geq 2$, i.e.

$$\dot{x}_i = x_i P_i(x_1, \dots, x_n),$$

where P_i are polynomials of degree 1. Finally in 1936 Kolmogorov [3] extended those systems to arbitrary degree, i.e. the polynomials P_i can have any degree. These last systems are now called *Kolmogorov systems*.

The Lotka-Volterra and Kolmogorov systems have been used for modelling many natural phenomena, such as the time evolution of conflicting species in biology [4], chemical reactions [5], plasma physics [6], hydrodynamics [7], and many other phenomena as social science and economics [8].

We want to study the limit cycles of the Kolmogorov systems of degree 3 in $\mathbb{R}^3$ which bifurcate in the zero-Hopf bifurcations of the singular points (a,b,c) which are not on the invariant planes $x=0$, $y=0$ and $z=0$ of the Kolmogorov

* Corresponding author.
E-mail addresses: erikadiz.pita@usc.es (É. Diz-Pita), jllibre@mat.uab.cat (J. Llibre), mvictoria.otero@usc.es (M.V. Otero-Espinar), cvalls@mat.tecnico.ulisboa.pt (C. Valls).

https://doi.org/10.1016/j.cnsns.2020.105621
1007-5704/© 2020 Elsevier B.V. All rights reserved.