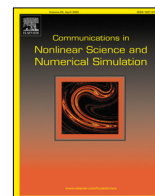




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Phase portraits of a family of Kolmogorov systems with infinitely many singular points at infinity



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ABSTRACT

We give the topological classification of the global phase portraits in the Poincaré disc of the Kolmogorov systems

$$\dot{x} = x(a_0 + c_1x + c_2z^2 + c_3z),$$

$$\dot{z} = z(c_0 + c_1x + c_2z^2 + c_3z),$$

which depend on five parameters and have infinitely many singular points at the infinity. We prove that these systems have 22 topologically distinct phase portraits.

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1. Introduction and statement of the main results

Kolmogorov systems are polynomial differential systems of the form $\dot{x}_i = x_i P_i(x_1, \dots, x_n)$ for $i = 1, \dots, n$, where P_i are polynomials. These include, for instance, Lotka–Volterra or May–Leonard systems. Kolmogorov systems can be used for modelling problems from different sciences as the evolution of competing species [1–5], plasma physics [6], hydrodynamics [7], chemical reactions [8], the study of black holes [9], and economic [10–12] or social problems, as the evolution of the number of internet users [13].

Recently, some works on the global dynamics of these systems have been carried out. For example, for the May–Leonard systems, which have the form

$$\dot{x} = x(1 - x - ay - bz),$$

$$\dot{y} = y(1 - bx - y - az),$$

$$\dot{z} = z(1 - ax - by - z),$$

their global dynamics on the Poincaré sphere when $a + b = 2$ or $a = b$ were studied in [14], and the case with $a + b = -1$ were studied in [15].

The global dynamics of some particular Lotka–Volterra systems on dimension three has also been described on the Poincaré sphere as in [16], where the authors give the global phase portraits of a system that appears in the study of black holes, or in [17] where the description of the global dynamics of a system previously proposed and studied in [18–20] is finally completed.

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