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Phase portraits of a family of Kolmogorov systems with infinitely many singular points at infinity

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ABSTRACT

We give the topological classification of the global phase portraits in the Poincaré disc of the Kolmogorov systems

$$\dot{x} = x\left(a_0 + c_1 x + c_2 z^2 + c_3 z\right), \qquad \dot{z} = z\left(c_0 + c_1 x + c_2 z^2 + c_3 z\right),$$

which depend on five parameters and have infinitely many singular points at the infinity. We prove that these systems have 22 topologically distinct phase portraits.

1. Introduction and statement of the main results

Kolmogorov systems are polynomial differential systems of the form $\dot{x}_i = x_i P_i(x_1, \ldots, x_n)$ for $i = 1, \ldots, n$, where P_i are polynomials. These include, for instance, Lotka–Volterra or May–Leonard systems. Kolmogorov systems can be used for modelling problems from different sciences as the evolution of competing species [1–5], plasma physics [6], hydrodynamics [7], chemical reactions [8], the study of black holes [9], and economic [10–12] or social problems, as the evolution of the number of internet users [13].

Recently, some works on the global dynamics of these systems have been carried out. For example, for the May–Leonard systems, which have the form

$$\dot{x} = x(1 - x - ay - bz), \qquad \dot{y} = y(1 - bx - y - az), \qquad \dot{z} = z(1 - ax - by - z),$$

their global dynamics on the Poincaré sphere when $a + b = 2$ or $a = b$ were studied in [14], and the case with $a + b = -1$ were studied in [15].

The global dynamics of some particular Lotka–Volterra systems on dimension three has also been described on the Poincaré sphere as in [16], where the authors give the global phase portraits of a system that appears in the study of black holes, or in [17] where the description of the global dynamics of a system previously proposed and studied in [18–20] is finally completed.

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