

Planar Kolmogorov systems with infinitely many singular points at infinity

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Received (to be inserted by publisher)

We classify the global dynamics of the five-parameter family of planar Kolmogorov systems

$$\begin{aligned}\dot{y} &= y(b_0 + b_1yz + b_2y + b_3z), \\ \dot{z} &= z(c_0 + b_1yz + b_2y + b_3z),\end{aligned}$$

which has been obtained from the Lotka-Volterra systems of dimension three. We give the topological classification of their phase portraits in the Poincaré disc, so we can describe the dynamics of these systems near infinity. We prove that these systems have 13 topologically distinct global phase portraits.

Keywords: Kolmogorov system, Lotka–Volterra system, phase portrait, Poincaré disc.

1. Introduction

Kolmogorov systems are differential systems of the form

$$\dot{x}_i = x_i P_i(x_1, \dots, x_n), \quad i = 1, \dots, n,$$

where P_i are polynomials. Particular cases of these systems are, for example, Lotka-Volterra systems. All of them have been used for modelling problems from different sciences as the interaction between species [Arnoedo *et al.*, 1980], [Coste *et al.*, 1979], [Llibre & Xiao, 2014], [Lois-Prados & Precup, 2020], [Smale, 1976], plasma physics [Laval & Pellat, 1975], hydrodynamics [Busse, 1981], chemical reactions [Hering, 1990] or economic and social problems [Gandolfo, 2009, 2008], [Wijeratne *et al.*, 2009].