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PHASE PORTRAITS OF A FAMILY OF KOLMOGOROV SYSTEMS DEPENDING ON SIX PARAMETERS

ÉRIKA DIZ-PITA, JAUME LLIBRE, M. VICTORIA OTERO-ESPINAR

Abstract. We consider a general 3-dimensional Lotka-Volterra system with a rational first integral of degree two of the form $H = x^i y^j z^k$. The restriction of this Lotka-Volterra system to each surface $H(x, y, z) = h$ varying $h \in \mathbb{R}$ provide Kolmogorov systems. With the additional assumption that they have a Darboux invariant of the form $x^\ell y^m e^{st}$ they reduce to the Kolmogorov systems

$$\dot{x} = x\big(a_0 - \mu(c_1 x + c_2 z^2 + c_3 z)\big),$$
$$\dot{z} = z\big(c_0 + c_1 x + c_2 z^2 + c_3 z\big).$$

We classify the phase portraits in the Poincaré disc of all these Kolmogorov systems which depend on six parameters.

1. Introduction

The Lotka-Volterra systems have been used for modeling many natural phenomena, such as the time evolution of conflicting species in biology [20], chemical reactions, plasma physics [15] or hydrodynamics [6], just as other problems from social science and economics.

These systems, which are polynomial differential equations of degree two, were initially proposed, independently, by Lotka in 1925 and Volterra in 1926, both in the context of competing species. Later on Lotka-Volterra systems were generalized and considered in arbitrary dimension, i.e.

$$\dot{x_i} = x_i\Big(a_{i0} + \sum_{j=1}^{n} a_{ij} x_j\Big), \quad i = 1, \dots, n.$$

Consequently the applications of these systems started to multiply. Moreover Kolmogorov in [14] extended the Lotka-Volterra systems as follows

$$\dot{x_i} = x_i P_i(x_1, \dots, x_n), \quad i = 1, \dots, n,$$

where P_i are polynomials of degre at most m. These kind of systems are now known as Kolmogorov systems. They have in particular all the applications of the Lotka-Volterra systems as for instance in the study of the black holes in cosmology, see [1].

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