Poincaré Compactification of the Collinear Three Body Problem

by

J. Delgado*, E. A. Lacomba *, J. Llibre** and E. Pérez*

*Departamento de Matemáticas, Universidad Autónoma Metropolitana–Iztapalapa, Apdo. 55534, México, D.F., México.

**Departament de Matemàtiques, Universitat Autònoma de Barcelona, 08193 Bellaterra, Spain.

1 Introduction

The vector field of the (Newtonian) n-body problem is defined on a non compact manifold. In the boundary of this manifold there are all the singularities due to collisions and to escapes or captures at infinity.

In this paper we describe a method to extend analytically the vector field of the n-body problem to a compact manifold, in fact to a sphere. This compactification technique consists in writing the equations of motion as a polynomial vector field and applying then the Poincaré compactification.

The vector field of the n-body problem can be written in polynomial form by using the ideas of Heggie [Heg] and other authors. Essentially these ideas consist in the regularization of all the binary collisions. For obtaining this polynomial vector field it is necessary (in general) to introduce redundant variables. Since any polynomial vector field in $\mathbf{R}^m$ can be extended analytically to the m-dimensional sphere (see [CL] or Section 2), we get an extension of the n-body problem to a compact manifold.

This method applied to the n-body problem has two disadvantages. First, it introduces redundant variables and second, the critical points of the compactified vector field are degenerate.

The main goal of this paper is to apply this technique to the collinear 3-body problem, analyzing the structure of the critical points of the compactified vector field.

This paper is organized as follows. In Section 2 we introduce the Poincaré compactification of a polynomial Hamiltonian vector field. Before studying the collinear 3-body problem we consider a simpler case, the rectilinear Kepler Problem (Section 3).

The study of the collinear 3-body problem is separated in two sections. In Section 4 we adapt the ideas of Heggie to write the equations of motion in a polynomial form, making their compactified extension. Finally, in Section 5 we study the structure of the set of critical points for the compactified collinear 3-body problem. Some of the results obtained in this paper are presented in [DLLP1].

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