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Periods of Morse–Smale diffeomorphisms on $\mathbb{S}^n$, $\mathbb{S}^m \times \mathbb{S}^n$, $\mathbb{C}\mathbf{P}^n$ and $\mathbb{H}\mathbf{P}^n$

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Abstract. We study the set of periods of the Morse–Smale diffeomorphisms on the n-dimensional sphere $\mathbb{S}^n$, on products of two spheres of arbitrary dimension $\mathbb{S}^m \times \mathbb{S}^n$ with $m \neq n$, on the n-dimensional complex projective space $\mathbb{C}\mathbf{P}^n$ and on the n-dimensional quaternion projective space $\mathbb{H}\mathbf{P}^n$. We classify the minimal sets of Lefschetz periods for such Morse–Smale diffeomorphisms. This characterization is done using the induced maps on the homology. The main tool used is the Lefschetz zeta function.

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1. Introduction

Understanding the periodic orbits and the set of periods of a map is a very important problem in dynamical systems. The Lefschetz numbers are one of the most useful tools to study the existence of fixed points and periodic orbits of self-maps on compact manifolds. In this paper, we obtain information on the set of periods of certain diffeomorphisms on compact manifolds using the Lefschetz zeta function, which is a generating function of the Lefschetz numbers of the iterates of a map.

Let M be a compact manifold, let $f : M \to M$ be a continuous map, and denote by f^m the mth iterate of f. A point $x \in M$ such that $f(x) = x$ is called a *fixed point*, or a *periodic point of period* 1 of f. A point $x \in M$ is called *periodic of period* $k > 1$ if $f^k(x) = x$ and $f^m(x) \neq x$ for all $m = 1, \dots, k-1$, and the set formed by the iterates of x, i.e., $\{x,\ f(x),\ \dots,\ f^{k-1}(x)\}$, is called the *periodic orbit* of the periodic point x.

As usual $\mathbb{N}$ denotes the set of all positive integers. Then $\mathrm{Per}(f)$ is the set $\{k \in \mathbb{N} : f \text{ has a periodic orbit of period } k\}$.

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