

## DIFFERENTIABLE INVARIANT MANIFOLDS OF NILPOTENT PARABOLIC POINTS

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**ABSTRACT.** We consider a map  $F$  of class  $C^r$  with a fixed point of parabolic type whose differential is not diagonalizable, and we study the existence and regularity of the invariant manifolds associated with the fixed point using the parameterization method. Concretely, we show that under suitable conditions on the coefficients of  $F$ , there exist invariant curves of class  $C^r$  away from the fixed point, and that they are analytic when  $F$  is analytic. The differentiability result is obtained as an application of the fiber contraction theorem. We also provide an algorithm to compute an approximation of a parameterization of the invariant curves and a normal form of the restricted dynamics of  $F$  on them.

**1. Introduction.** Invariant manifolds play a central role in the study of dynamical systems. There is a huge amount of literature devoted to study them in many different settings. In this paper we deal with the invariant manifolds of a type of parabolic fixed points in dimension two.

Parabolic points appear generically in two-parameter families of planar maps or in one-parameter ones in the case of area-preserving maps. In particular they appear when a family of maps undergoes a Bogdanov-Takens bifurcation [6, 27].

In some problems in Celestial Mechanics it is useful to consider parabolic points or parabolic orbits at infinity in order to use their invariant manifolds (provided they exist) to study features of the dynamics in the finite phase space. The local study in a neighborhood of such points is done by means of a change of variables

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