LINEARIZATION OF TOPOLOGICALLY ANOSOV HOMEOMORPHISMS OF NON COMPACT SURFACES.

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Abstract. We study the dynamics of *Topologically Anosov* homeomorphisms of non compact surfaces. In the case of surfaces of genus zero and finite type, we classify them. We prove that if $f : S \to S$, is a Topologically Anosov homeomorphism where S is a non-compact surface of genus zero and finite type, then $S = \mathbb{R}^2$ and f is conjugate to a homothety or reverse homothety (depending on wether f preserves or reverses orientation). A weaker version of this result was conjectured in [CGX].

1. Introduction

Let $f : S \to S$ be a homeomorphism and $\delta : S \to \mathbb{R}$ a continuous and strictly positive function. A *δ-pseudo-orbit* for f is a sequence $(x_n)_{n\in\mathbb{Z}} \subset S$ such that $d(f(x_n), x_{n+1}) < \delta(f(x_n))$ for all $n \in \mathbb{Z}$. If $\epsilon : S \to \mathbb{R}$ a continuous and strictly positive function, then a δ-pseudo-orbit $(x_n)_{n\in\mathbb{N}}$ is *ϵ-shadowed* by an orbit, if there exists $x \in S$ such that $d(x_n, f^n(x)) < \epsilon(x_n)$ for all $n \in \mathbb{Z}$.

Throughout this paper $f : S \to S$ is a *Topologically Anosov* (TA) homeomorphism. That is:

- it is topologically expansive: there exists a continuous and strictly positive function $\epsilon : S \to \mathbb{R}$ such that for all $x, y \in S, x \neq y$ there exists $k \in \mathbb{Z}$ satisfying $d(f^k(x), f^k(y)) > \epsilon(f^k(x))$;
- it satisfies the *topological shadowing property*: for all continuous and strictly positive function $\epsilon : S \to \mathbb{R}$ there exists $\delta : S \to \mathbb{R}$ a continuous and strictly positive function such that every δ-pseudo-orbit is ϵ-shadowed by an orbit.

These definitions are generalizations of the classic notions of *uniform* expansivity and pseudo-orbit tracing property, suited for non-compact metric spaces. On non-compact spaces it is well known that a dynamical system may be expansive or have the shadowing property with respect to one metric, but not with respect to another metric that induces the same topology. Topological definitions of expansiveness and shadowing were given in [DLRW] for first countable, locally compact, paracompact and Hausdorff topological spaces; they are equivalent to the usual metric definitions for homeomorphisms on compact metric spaces, but are independent of any change of compatible metric. The definitions we gave correspond to those given in [DLRW] in the case of metric spaces, and appeared fist in literature in [LNY].

To illustrate what happens in the non-compact setting, note that a rigid translation in the plane is topologically expansive but does not satisfy the topological shadowing property. An example of TA homeomorphism is any homothety (or reverse homothety) in $\mathbb{R}^2$ (see [C] for a proof). As being TA is a conjugacy invariant, the whole conjugacy class of homotheties belongs to the family of TA homeomorphisms. In this work we deal with the problem of classifying TA homeomorphisms.