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On the Existence of Symmetric Bicircular Central Configurations of the 3*n*-Body Problem

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Abstract

In this paper, we consider central configurations of the planar $3n$-body problem consisting of n masses at the vertices of a regular n-gon inscribed in a circle of radius r and $2n$ masses at the vertices of a second (not necessarily regular) concentric $2n$-gon inscribed in a circle of radius ar which are symmetric in the sense that the set of positions of the $3n$ masses and the set of the corresponding masses are invariant under the action of a finite subgroup of $O(2)$. There are two different types of such configurations. In the first type, called *regular bicircular central configurations of the* $3n$*-body problem*, the second $2n$-gon is regular, n of the vertices of the second n-gon are aligned with the vertices of the first regular n-gon and the masses at the vertices of this $2n$-gon alternate values. In the second type, called *semiregular bicircular central configurations of the* $3n$*-body problem*, the second $2n$-gon is semiregular and the masses at its vertices are all of them equal. A semiregular $2n$-gon has n pair of vertices symmetric by a reflection of an angle β with respect to the axis of symmetry of the first regular n-gon. Our aim is to analyze the set of values of the parameter a for the regular $2n$-gon and of the parameters (a, β) for the semiregular $2n$-gon providing symmetric bicircular central configurations. In particular, for all $n \geq 2$ we prove analytically the existence of symmetric bicircular central configurations with a (respectively (a, β)) satisfying some particular conditions. Using either computer-assisted results or numerical results, we also describe the complete set of values of a (respectively (a, β)) providing symmetric bicircular central configurations for $n = 2, 3, 4, 5$ and we give numerical evidences that the pattern for $n > 5$ is the same as the one for $n = 5$.

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