PLANAR CENTRAL CONFIGURATIONS OF SOME RESTRICTED $(4+1)$-BODY PROBLEMS

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Abstract. We start with the 13 central configurations of the restricted $(4+1)$–problem where the four primaries have equal masses and are located at the vertices of a square. Then we describe the evolution of these central configurations when some of the masses of the four primaries tend to zero and the remainder ones keep constant. More precisely, we consider the cases where one of the masses tends to zero, where either two adjacent or two opposite equal masses tend to zero simultaneously, and where three equal masses tend to zero simultaneously. Here simultaneously means that the masses which go to zero take the same value at any moment.

1. Introduction and statement of the main results

The *n-body problem* is the problem of studying the motions of n punctual masses interacting between them under the Newtonian gravitation.

A configuration of the bodies of the n-body problem is called *central* when the acceleration of each body is proportional to the position vector of the body with respect to the center of mass.

The set of all planar central configurations is invariant under homotheties with respect to the center of mass and rotations. When we count the number of central configurations we mean the number of equivalence classes with respect the equivalence relations defined by these homotheties and rotations.

Central configurations are important in the analysis of the n-body problem for several reasons, here we only mentioned briefly some of them.

(1) They allow to compute all the homographic solutions of this problem (see [15]).
(2) Every motion starting or ending in a total collision is asymptotic to a central configuration (see [4, 10]).
(3) Every parabolic motion of the n bodies is asymptotic to a central configuration (see [4, 10]).
(4) They play a role in the study of the invariant sets obtained fixing the energy and the angular momentum (see [12, 13]).
(5) They have been used for different missions in the solar system (see [6, 7].

The central configurations of the 2– and 3-body problem are known (see [9]), but the problem of finding the central configurations when $n > 3$ is far to be solved. More precisely, for $n > 3$ we only know the central configurations for some particular n-body problems where, in general, the configurations satisfy some geometrical properties, or some of the masses are equal.

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