PROBABILITY OF EXISTENCE OF LIMIT CYCLES FOR A FAMILY OF PLANAR SYSTEMS

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Abstract. The goal of this work is the study of the probability of occurrence of limit cycles for a family of planar differential systems that are a natural extension of linear ones. To prove our results we first develop several results of non-existence, existence, uniqueness and non-uniqueness of limit cycles for this family. They are obtained by studying some Abelian integrals, via degenerate Adronov-Hopf bifurcations or by using the Bendixson-Dulac criterion. To the best of our knowledge, this is the first time that the probability of existence of limit cycles for a non-trivial family of planar systems is obtained analytically. In particular, we give vector fields for which the probability of having limit cycles is positive, but as small as desired.

1. Introduction and main results

Many efforts have been devoted to study the existence, non-existence, uniqueness or number of limit cycles of planar autonomous systems, see for instance [8, 19, 22, 27, 28] and their references. Many of these results involve polynomial differential systems due to the big interest on the celebrated Hilbert's XVI-th problem. Nevertheless, as far as the authors know, the problem of knowing which is the probability of existence of limit cycles for a given family of such vector fields has been seldom analytically studied. In this work we propose a quite natural family of planar vector fields for which we face this problem. Before stating our results we introduce and formalize this question in more detail.

Consider the planar linear differential systems

$$\dot{x} = ax + by, \quad \dot{y} = cx + dy, \tag{1}$$

where $(a, b, c, d) \in \mathbb{R}^4$. To know the probability of occurrence of each of its possible phase portraits (saddle, node, focus, center, ...) there is a well established way to mathematize this problem. Take the planar random linear systems

$$\dot{x} = Ax + By, \quad \dot{y} = Cx + Dy,$$

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