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Some Theorems on the Existence, Uniqueness, and Nonexistence of Limit Cycles for Quadratic Systems

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Given a quadratic system (QS) with a focus or a center at the origin we write it in the form $\dot{x} = y + P_2(x, y)$, $\dot{y} = -x + dy + Q_2(x, y)$ where P_2 and Q_2 are homogeneous polynomials of degree 2. If we define $F(x, y) = (x - dy) P_2(x, y) + yQ_2(x, y)$ and $g(x, y) = xQ_2(x, y) - yP_2(x, y)$ we give results of existence, nonexistence, and uniqueness of limit cycles if $F(x, y)\, g(x, y)$ does not change of sign. Then, by using these results plus the properties on the evolution of the limit cycles of the semicomplete families of rotated vector fields we can study some particular families of QS, i.e., the QS with a unique finite singularity and the bounded QS with either one or two finite singularities.

1. INTRODUCTION AND STATEMENT OF THE MAIN RESULTS

We consider the differential system $\dot{x} = dx/dt = P(x, y)$, $\dot{y} = dy/dt = Q(x, y)$ where P and Q are polynomials of second degree with real constant coefficients, and x, y, and t are also real. When the maximum $\{$degree P, degree $Q\} = 2$ we call such systems *quadratic systems*, QS, for abbreviation. Any QS on the plane with a focus or a center at the origin can be written in the form

$$\dot{x} = y + P_2(x, y), \qquad \dot{y} = -x + dy + Q_2(x, y) \text{ with } |d| < 2, \tag{1}$$

where $P_2(x, y)$ and $Q_2(x, y)$ are homogeneous polynomials of degree two.

We define two functions associated to the differential equation (1),

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