ON THE CONFIGURATIONS OF LIMIT CYCLES FOR POLYNOMIAL VECTOR FIELDS IN THE PLANE

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Abstract In the set of limit cycles of a polynomial vector field $X = (P, Q)$ in the plane two equivalence relations are defined Their classes of equivalence introduce the notions of fan and nest of limit cycles By using them we obtain information about the configurations of limit cycles of X taking into account the degrees of the polynomials P and Q

Hilbert's sixteenth problem asks for the maximum number and position of limit cycles for a differential system of the form

$$\dot{x} = P(x,y) \ , \ \dot{y} = Q(x,y), \tag{0.1}$$

where P and Q are polynomials of degree n in x and y .

Recall that a solution of (0.1) is a *limit cycle* if it is an isolated periodic orbit. A more precise statement of the Hilbert's sixteenth problem is to find the bound of the number of limit cycles of system (0.1) with degree $(P) = n$, degree$(Q) = m$ and not necessarily $n = m$.

Let $\mathcal{X}_{n,m}$ be the set of real polynomial vector fields $X = (P, Q)$ defined in $\mathbf{R}^2$ such that the degree of P and Q are n and m, respectively. Let $G_{n,m}$ be the set of all $X \in \mathcal{X}_{n,m}$ such that

$$Card\,\{a \in \mathbf{C}^2 : P(a) = 0\,,\, Q(a) = 0\,\} = \ degree(P)\, \cdot \, degree(Q).$$

It is not hard to see that $\mathcal{X}_{n,m} \setminus G_{n,m}$ is contained in an algebraic hypersurface of $\mathcal{X}_{n,m}$ (for more details see [**CL1**]).

Let $X = (P, Q)$ be the polynomial vector field associated to the system (0.1). If C is a limit cycle, we denote by *Int*(C) the region bounded by C. The set $s(C)$ is formed by the critical points of X which are in Int(C) while $s_+(C)$ denotes the critical points of $s(C)$ with index 1.