JOURNAL OF DIFFERENTIAL EQUATIONS **82**, 71–97 (1989)

Configurations of Fans and Nests of Limit Cycles for Polynomial Vector Fields in the Plane

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Received August 17, 1988

In the set of limit cycles of a polynomial vector field $X = (P, Q)$ in the plane two equivalence relations are defined. Their classes of equivalence introduce the notions of fan and nest of limit cycles. By using them we obtain information about the configurations of limit cycles of X taking into account the degrees of the polynomials P and Q.

0. INTRODUCTION

Hilbert's sixteenth problem asks for the maximum number and position of limit cycles for a differential system of the form

$$\dot{x} = P(x, y), \qquad \dot{y} = Q(x, y), \tag{0.1}$$

where P and Q are polynomials of degree n in x and y.

Recall that a solution of (0.1) is a *limit cycle* if it is an isolated periodic orbit. A more precise statement of Hilbert's sixteenth problem is to find the bound of the number of limit cycles of system (0.1) with degree $(P) = n$, degree$(Q) = m$, and not necessarily $n = m$.

Let $\mathscr{X}_{n,m}$ be the set of real polynomial vector fields $X = (P, Q)$ defined in $\mathbf{R}^2$ such that the degrees of P and Q are n and m, respectively. Let $G_{n,m}$ be the set all $X \in \mathscr{X}_{n,m}$ such that

$$\text{Card}\{a \in \mathbf{C}^2 : P(a) = 0,\ Q(a) = 0\} = \text{degree}(P) \cdot \text{degree}(Q).$$

0022-0396/89 $3.00