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ASYMPTOTIC STABILITY FOR BLOCK TRIANGULAR MAPS

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Dedicated to Professor Mustafa R. S. Kulenović on the occassion of his 70th anniversary

ABSTRACT. We prove a result concerning the asymptotic stability and the basin of attraction of fixed points for block triangular maps in $\mathbb{R}^n$. This result is applied to some families of discrete dynamical systems and several types of difference equations.

1. INTRODUCTION AND MAIN RESULTS

In this work we consider block triangular discrete dynamical systems (DDS) of the form

$$\begin{cases} x_{n+1} &= f(x_n,u_n), \\ u_{n+1} &= g(u_n), \end{cases} \tag{1.1}$$

where $x \in \mathbb{R}^m$, $u \in \mathbb{R}^k$ and $f : \mathbb{R}^m \times \mathbb{R}^k \to \mathbb{R}^m$, $g : \mathbb{R}^k \to \mathbb{R}^k$, and m and k are positive integers. We assume that there exists a locally attracting fiber $\{u = u_*\}$, which is invariant and with a unique fixed point on it. The existence of this locally attracting fiber is equivalent to the existence of a locally asymptotically stable fixed point of the subsystem $u_{n+1} = g(u_n)$. We will assume that on this limit fiber the dynamics given by the map $x \to f(x,u_*)$ has a globally asymptotically stable (GAS) fixed point. The problem considered here is to give conditions on the map f under which this fact forces the same behavior for all initial conditions in the whole basin of attraction of this fiber, that is (x_*,u_*) is also GAS for the DDS (1.1) on this basin. The next theorem is our main result. In this work $||y||$ denotes any vector norm of $y \in \mathbb{R}^\ell$.

Theorem 1.1. *Consider the DDS* (1.1) *with f and g continuous and such that:*

(a) *The map f is sublinear in x, that is, there exist continuous functions $M,N : \mathbb{R}^k \to \mathbb{R}^+ \cup \{0\}$ such that*

$$||f(x,u)|| \le M(u) + N(u)||x||. \tag{1.2}$$

(b) *The point $u = u_*$ is a stable attractor for the DDS $u_{n+1} = g(u_n)$.*

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