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Nilpotent Center in a Continuous Piecewise Quadratic Polynomial Hamiltonian Vector Field

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In this paper, we study the global dynamics of continuous piecewise quadratic Hamiltonian systems separated by the straight line $x = 0$, where these kinds of systems have a nilpotent center at $(0, 0)$, which comes from the combination of two cusps of both Hamiltonian systems. By the Poincaré compactification we classify the global phase portraits of these systems. We must mention that it is extremely rare to find works studying the center-focus problem in piecewise smooth systems with nonelementary singular points as we did here.

Keywords: Nilpotent; center; Hamiltonian; phase portrait; piecewise smooth system.

1. Introduction and Statement of the Main Results

By the works of Poincaré [Poincaré, 1881] and Dulac [Dulac, 1908] the center-focus problem, i.e the problem of distinguishing when a singular point is either a focus or a center, has been one of the main problems in the qualitative theory of the planar differential equations. In the planar polynomial vector fields a singular point is either elementary or nonelementary. The linear part of the elementary singular point has at least one nonzero eigenvalue, and the nonelementary one has two zero eigenvalues. And a nonelementary singular point is called nilpotent if its linear part is not identically zero, otherwise it is called degenerate. Moreover, if a planar polynomial differential equation has a *linear type center*, a *nilpotent center* and a *degenerate center* at the origin, after affine change of time and variables, this differential equation can be written as

$$(\dot{x}, \dot{y}) = \left\{ \begin{matrix} (-y, x) \\ (y, 0) \\ (0, 0) \end{matrix} \right\} + (F(x, y), G(x, y)), \qquad (1)$$

respectively, where $F(x, y)$ and $G(x, y)$ are real polynomials without constant and linear terms.

The center-focus problem in the quadratic polynomial differential equations has been solved, see [Artés & Llibre, 1994; Bautin, 1952; Kapteyn, 1911, 1912; Schlomiuk, 1993; Żołądek, 1994a]. For the cubic polynomial differential equations there are partial works in the center-focus problem, see [Aziz *et al.*, 2014; Cima & Llibre, 1990; Malkin, 1964; Vulpe & Sibirskii, 1988; Żołądek, 1994b, 1996] for an

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