International Journal of Bifurcation and Chaos, Vol. 33, No. 6 (2023) 2350075 (13 pages)

DOI: 10.1142/S021812742350075X

Nonexistence and Uniqueness of Limit Cycles in a Class of Three-Dimensional Piecewise Linear Differential Systems

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Received January 10, 2023; Revised March 18, 2023

During the last twenty years there has been increasing interest in studying the piecewise differential systems, mainly due to their many applications in natural science and technology. Up to now the most studied differential systems are in dimension two, here we study them in dimension three. One of the main difficulties for studying these differential systems consists in controlling the existence and nonexistence of limit cycles, and the numbers when they exist.

In this paper, we study the nonsymmetric limit cycles for a family of three-dimensional piecewise linear differential systems with three zones separated by two parallel planes. For this class of differential systems we study the nonexistence, existence and uniqueness of their limit cycles.

Keywords: Limit cycle; periodic orbit; three-dimensional; piecewise linear differential system.

1. Introduction and Statement of the Main Results

As we know, the continuous and discontinuous piecewise smooth differential systems play an important role inside many disciplines, such as control theory, electrical engineering, mechanics, biology and economics, see for instance the papers [Andronov *et al.*, 1966; Coombes, 2008; Lefschetz, 1965; Leine & Nijmeijer, 2004; Kunze & Kupper, 1997] and the references quoted there.

The maximum number of *limit cycles*, i.e. the periodic orbits isolated in the set of all periodic orbits, for differential systems is the second part of Hilbert's 16th problem. In the last decades, they have been extensively studied on the limit cycles of continuous and discontinuous piecewise differential systems in $\mathbb{R}^2$, see [Braga & Mello, 2013; Buzzi *et al.*, 2013; Chen *et al.*, 2021; Freire *et al.*, 1998; Freire *et al.*, 2012, 2014; Giannakopoulos & Pliete, 2014; Han & Zhang, 2010; Huan & Yang, 2013;

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