LEFSCHETZ ZETA FUNCTION AND TOPOLOGICAL ENTROPY

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Abstract. In this paper we study the dynamical consequences of simple algebraic properties of the Lefschetz zeta function $Z_f(t)$ associated to a continuous self-map $f : M \to M$ of a compact manifold M, which is always a rational function $P(t)/Q(t)$. We show that there is a relation between the parity of the degrees of $P(t)$ and $Q(t)$ and the finiteness of the set of periodic points of f on one hand, and vanishing topological entropy on the other (Theorems 5 and 6).

Given a continuous self-map f of a compact manifold M of dimension n, its *Lefschetz number* is defined as

$$L(f) = \sum_{k=0}^{n} (-1)^k \mathrm{tr}\ (f_{*k}),$$

where $f_{*k} : H_k(M; \mathbb{Q}) \to H_k(M; \mathbb{Q})$ is the endomorphism induced by f on the k-th rational homology group of M. The Lefschetz fixed point theorem says that if $L(f) \neq 0$ then f has a fixed point. For the purpose of studying the whole set of periodic points of f, it is useful to consider the *Lefschetz zeta function*

$$Z_f(t) = \exp\left(\sum_{m=1}^{\infty} \frac{L(f^m)}{m} t^m \right).$$

which is a generating function for the Lefschetz numbers of all iterates of f and can be computed from the homological endomorphisms f_{*k} of f as follows

$$Z_f(t) = \prod_{k=0}^{n} \det\ (I_{j_k} - t f_{*k})^{(-1)^{k+1}}, \tag{1}$$

where $j_k = \dim_{\mathbb{Q}} H_k(M; \mathbb{Q})$. see [F1] for more details.

The following theorem is due to Fried (see Theorem 6 of [F2]).