PERIODS FOR TRANSVERSAL MAPS OF SPHERES

JOSEFINA CASASAYAS*, JAUME LLIBRE** AND ANA NUNES+

* Departament de Matemàtica Aplicada i Anàlisi, Universitat de Barcelona, Gran Via 585, 08071 Barcelona, Spain.

** Departament de Matemàtiques, Universitat Autònoma de Barcelona, Bellaterra, 08193 Barcelona, Spain.

+ Centre de Recerca Matemàtica, Universitat Autònoma de Barcelona, Bellaterra, 08193 Barcelona, Spain, on leave from Departamento de Física, Universidade de Lisboa, Campo Grande, Ed C1, Piso 4, 1700 Lisboa, Portugal.

Abstract. A result is given which relates the set of periods for a transversal self-map of a sphere with its degree through the associated Lefschetz zeta function. Our hypothesis on the map are more restrictive than in the case when one uses Nielsen theory, but we are able to obtain more information, namely concerning the stability type of the periodic orbits of the map.

Given a continuous self-map of a compact manifold M of dimension n, its *Lefschetz number* is defined as

$$L(f) = \sum_{k=0}^{n} (-1)^k \operatorname{tr}(f_{*k}),$$

where $f_{*k} : H_k(M;\mathbb{Q}) \to H_k(M;\mathbb{Q})$ is the endomorphism induced by f on the k-th rational homology group of M. The Lefschetz fixed point theorem says that if $L(f) \neq 0$ then f has a fixed point. For the purpose of studying the set

$$\operatorname{Per}(f) = \{m \in \mathbb{N} : f \text{ has a periodic orbit of minimal period } m\}.$$

it is useful to consider the *Lefschetz zeta function*

$$Z_f(t) = \exp\left(\sum_{m=1}^{\infty} \frac{L(f^m)}{m} t^m\right).$$