SYMMETRIC PERIODIC ORBITS IN THE ANISOTROPIC KEPLER PROBLEM

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ABSTRACT. The anisotropic Kepler problem has a group of symmetries with three generators; they are symmetries respect to zero velocity curve and the two axes of motion's plane. For a fixed negative energy level it has four homothetic orbits. We describe the symmetric periodic orbits near these homothetic orbits. Full details and proofs will appear elsewhere (Casasayas-Llibre).

1. INTRODUCTION AND EQUATIONS OF MOTION.

The anisotropic Kepler problem was introduced by Gutzwiller (1973) to model certain quantum mechanical systems. But for us it has a mathematical interest because it is an easy model in order to study usual tools in the analysis of the n-body problem as non-integrability, collision manifold,... (Devaney, 1981).

This problem deals with the motion of a body which is attracted by a gravitational potential and has an anisotropic mass. It is described by the Hamiltonian system

$$\dot{q} = M^{-1}p, \qquad \dot{p} = -\nabla V(q), \tag{1}$$

where

$$q = (q_1, q_2) \in R^2 - \{(0,0)\} \text{ and } p = (p_1, p_2) \in R^2$$

are the position and momentum coordinates of the body,

$$M^{-1} = \begin{pmatrix} \mu & 0 \\ 0 & 1 \end{pmatrix},$$

is the masses matrix and μ, $1 \leq \mu \leq +\infty$, is the mass parameter and

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