



On the basins of attraction of a one-dimensional family of root finding algorithms: from Newton to Traub

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Abstract

In this paper we study the dynamics of damped Traub's methods T_δ when applied to polynomials. The family of damped Traub's methods consists of root finding algorithms which contain both Newton's ($\delta = 0$) and Traub's method ($\delta = 1$). Our goal is to obtain several topological properties of the basins of attraction of the roots of a polynomial p under T_1 , which are used to determine a (universal) set of initial conditions for which convergence to all roots of p can be guaranteed. We also numerically explore the global properties of the dynamical plane for T_δ to better understand the connection between Newton's method and Traub's method.

Keywords Holomorphic dynamics · Julia and Fatou sets · Basins of attraction · Root finding algorithms · Simple connectivity · Unboundedness

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