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Crossing limit cycles of planar discontinuous piecewise differential systems formed by isochronous centres

Claudio A. Buzzi [a], Yagor Romano Carvalho [b] and Jaume Llibre [c]

[a]Mathematics Department, Universidade Estadual Paulista Julio de Mesquita Filho, São José do Rio Preto, São Paulo, Brazil; [b]Mathematics Department, Universidade de São Paulo, São Carlos, São Paulo, Brazil; [c]Mathematics Department, Universitat Autònoma de Barcelona, Barcelona, Spain

ABSTRACT

These last years an increasing interest appeared in studying the planar discontinuous piecewise differential systems motivated by the rich applications in modelling real phenomena. The understanding of the dynamics of these systems has many difficulties. One of them is the study of their limit cycles. In this paper, we study the maximum number of crossing limit cycles of some classes of planar discontinuous piecewise differential systems separated by a straight line and formed by combinations of linear centres (consequently isochronous) and cubic isochronous centres with homogeneous nonlinearities. For these classes of planar discontinuous piecewise differential systems we solved the extension of the 16th Hilbert problem, i.e. we provide an upper bound for their maximum number of crossing limit cycles.

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1. Introduction and statement of the main results

A *limit cycle* is a periodic orbit of a differential system in $\mathbb{R}^2$ that is isolated in the set of all its periodic orbits. The analysis of the existence of limit cycles became important in the applications of the real world because many phenomena are related to their existence, see for instance the Van der Pol oscillator [28, 29]. The study of limit cycles began with Poincaré [23] at the end of the nineteenth century. On the other hand, the study of the continuous piecewise linear differential systems separated by a straight line has special attention from the mathematicians, mainly because these systems appear in a natural way in the control theory, see for instance the books [10, 16, 20, 21], in mechanics, electrical circuits, economy, see for instance the books [1, 25] and the surveys [19, 26].

The easiest continuous piecewise linear differential systems are formed by two linear differential systems separated by a straight line and it is known that such systems have at most one limit cycle, see [7, 13, 17, 18]. But it is also known that if both linear differential systems are linear centres, then the continuous piecewise linear differential system has no limit cycles, see for example [15]. However, if we eliminate the continuity of such systems, that is, they do not need to coincide on the line of discontinuity, then it is known that these

CONTACT Yagor Romano Carvalho yagor.carvalho@usp.br