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Transcritical bifurcation at infinity in planar piecewise polynomial differential systems with two zones

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ABSTRACT
We present a general mechanism of generation of limit cycles in planar piecewise polynomial differential systems with two zones by means of a transcritical bifurcation at infinity and from a global centre. This study justifies the existence of limit cycles that arise through the intersection of the separation boundary with the one that characterizes the global centre.

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1. Introduction and statement of the main results

In the classical Qualitative Theory of Differential Equations, it is usual to study the global behaviour of the phase portrait of a given planar polynomial differential system by means of the Poincaré compactification [11]. When we apply this construction to a polynomial vector field G on $\mathbb{R}^2$, we obtain a new vector field on $\mathbb{S}^2\backslash\mathbb{S}^1$ through the central projections and its extension $\mathscr{P}(G)$ to the Poincaré sphere $\mathbb{S}^2$ is everywhere analytic and analytically equivalent to G in each hemisphere.

The vector field $\mathscr{P}(G)$, called *Poincaré compactification* of G, has the equator $\mathbb{S}^1$ as an invariant set which can be either a periodic orbit, a connected union of singular points and arcs of $\mathbb{S}^1$, or even foliated by singular points. In addition, if $\mathbb{S}^1$ is a periodic orbit then it cannot be a semistable one since the central projections provide two identical copies of the dynamics of the vector field G each of them on one hemisphere of $\mathbb{S}^2$.

By means of another projection, for instance, the gnomonic projection such as in [11], we can study the vector field $\mathscr{D}(G)$ obtained by the projection of $\mathscr{P}(G)$ onto $\mathbb{D}$, where $\mathbb{D} = \{(x, y) \in \mathbb{R}^2 : x^2 + y^2 \leq 1\}$ is the *Poincaré disc*. It follows that there exists a one-to-one correspondence between points placed at infinity of G and points on $\partial\mathbb{D} = \{(x, y) \in \mathbb{R}^2 : x^2 + y^2 = 1\}$ of $\mathscr{D}(G)$. In this sense, we say $p \in \partial\mathbb{D}$ is a singular point at infinity of the vector field G, if $\mathscr{D}(G)(p) = 0$. When G has no singular points at infinity we say G has a periodic orbit at infinity which is identified with $\partial\mathbb{D}$.

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