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ORIGINAL RESEARCH

On the Connection Between Global Centers and Global Injectivity in the Plane

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Abstract

In this note we revisit a result of Sabatini relating global injectivity of polynomial maps to global centers in the plane. We deliver a generalization of this result for C^2 maps defined on connected sets. The shape of the image is taking into account. Here we do not use Hadamard's invertibility theorem.

Keywords Centers · Global injectivity · Real Jacobian conjecture

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Introduction and Statement of the Main Results

Throughout our exposition $U \subset \mathbb{R}^2$ will be an open connected set.

Let $X, Y : U \to \mathbb{R}$ be C^k functions for some $k \in \mathbb{N}$. We consider the vector field $\mathcal{X} = (X, Y)$, or equivalently the system of differential equations

$$\dot{x} = X(x, y), \quad \dot{y} = Y(x, y). \tag{1}$$

Let z_0 be an isolated singular point of the system (1). We say that z_0 is a *center* of (1) when there exists a neighborhood V of z_0, $V \subset U$, such that each orbit of (1) in $V \setminus \{z_0\}$ is periodic. We define the *period annulus* of the center z_0, denoted by $\mathcal{P}_{z_0}$, as the maximal open connected set $W \subset U$ such that $W \setminus \{z_0\}$ is filled with periodic orbits of $\mathcal{X}$. We say that the center is *global* when $\mathcal{P}_{z_0} = U$. We say that the center is *isochronous* when the orbits in $\mathcal{P}_{z_0}$ have the same period.

When the singular point z_0 is non-degenerate, i.e. the determinant of the linear part of $\mathcal{X}$ in z_0 is different from zero, it is known that in order to have a center it is necessary

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