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LINEAR TYPE GLOBAL CENTERS OF CUBIC HAMILTONIAN SYSTEMS SYMMETRIC WITH RESPECT TO THE x-AXIS

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ABSTRACT. A polynomial differential system of degree 2 has no global centers (that is, centers defined in all the plane except the fixed point). In this paper we characterize the global centers of cubic Hamiltonian systems symmetric with respect to the x-axis, and such that the center has purely imaginary eigenvalues.

1. Introduction and statement of the results

The notion of center goes back to Poincaré and Dulac, see [4, 10]. They defined a *center* for a vector field on the real plane as a singular point having a neighborhood filled of periodic orbits with the exception of the singular point. The problem of distinguishing when a monodromic singular point is a focus or a center, known as the focus-center problem started precisely with Poincaré and Dulac and is still active nowadays with many questions still unsolved.

If an analytic system has a center, then it is known that after an affine change of variables and a rescaling of the time variable, it can be written in one of the following three forms:

$$\dot{x} = -y + P(x, y), \quad \dot{y} = x + Q(x, y),$$

called *linear type center*, which has a pair of purely imaginary eigenvalues;

$$\dot{x} = y + P(x, y), \quad \dot{y} = Q(x, y)$$

called *nilpotent center*; and

$$\dot{x} = P(x, y), \quad \dot{y} = Q(x, y)$$

called *degenerated center*, where $P(x, y)$ and $Q(x, y)$ are real analytic functions without constant and linear terms defined in a neighborhood of the origin.

We recall that a *global center* for a vector field on the plane is a singular point p having $\mathbb{R}^2$ filled of periodic orbits with the exception of the singular point. The easiest global center is the linear center $\dot{x} = -y$, $\dot{y} = x$. It is known (see [11, 1]) that quadratic polynomial differential systems have no global centers. The global degenerated (or homogeneous) centers were characterized in [**?**] while the global quasihomogeneous centers were studied in [5]. However the characterization of the global centers in the cases that the center is either nilpotent or a linear-type center

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