Fatou components and singularities of meromorphic functions

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We prove several results concerning the relative position of points in the postsingular set $P(f)$ of a meromorphic map f and the boundary of a Baker domain or the successive iterates of a wandering component. For Baker domains we answer a question of Mihaljević-Brandt and Rempe-Gillen. For wandering domains we show that if the iterates U_n of such a domain have uniformly bounded diameter, then there exists a sequence of postsingular values p_n such that $\operatorname{dist}(p_n, U_n) \to 0$ as $n \to \infty$. We also prove that if $U_n \cap P(f) = \emptyset$ and the postsingular set of f lies at a positive distance from the Julia set (in $\mathbb{C}$), then the sequence of iterates of any wandering domain must contain arbitrarily large disks. This allows to exclude the existence of wandering domains for some meromorphic maps with infinitely many poles and unbounded set of singular values.

1. Introduction and statement of the results

We consider dynamical systems defined by the iteration of a meromorphic function

$$f : \mathbb{C} \to \widehat{\mathbb{C}},$$

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