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ORIGINAL PAPER

The extended 16th Hilbert problem for a class of discontinuous piecewise differential systems

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Abstract In order to understand the dynamics of the planar differential systems, the limit cycles play a main role, but in general their study is not easy. These last years, an increasing interest appeared for studying the limit cycles of some classes of piecewise differential systems, due to the rich applications of this kind of differential systems. This paper solves the extended 16th Hilbert problem for a family of discontinuous planar differential systems with two regions separated by the straight line $x = 0$. By using the first integrals, we prove that the maximum number of crossing limit cycles in the family of systems formed by a linear center and a class of Hamiltonian isochronous global center with a polynomial first integral of degree $2n$ is 5.

Keywords Limit cycle · Discontinuous piecewise linear differential systems · Linear differential center · Hamiltonian isochronous global center

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1 Introduction

In this paper, we deal with the planar discontinuous piecewise differential systems defined by

$$\dot{x} = \mathbf{F}(\mathbf{x}) = \begin{cases} \mathbf{F}^-(\mathbf{x}) = (F_1^-(\mathbf{x}), F_2^-(\mathbf{x}))^T & \mathbf{x} \in \Sigma^-, \\ \mathbf{F}^+(\mathbf{x}) = (F_1^+(\mathbf{x}), F_2^+(\mathbf{x}))^T & \mathbf{x} \in \Sigma^+ \end{cases} \tag{1}$$

with $\mathbf{x} = (x, y)$ where Σ^- and Σ^+ are two regions in the plane defined by

$$\Sigma^- = \{(x, y) : x \leq 0\}, \quad \text{and} \quad \Sigma^+ = \{(x, y) : x \geq 0\}.$$

The set $\Sigma = \{(x, y) : x = 0\}$ is called the separation line of the plane.

In general, to find an upper bound for the maximum number of limit cycles for this class of systems has remained an open problem until now; this problem is known as the extended 16th Hilbert problem, which states to find an upper bound for the maximum number of limit cycles that the class of polynomial differential systems of a given degree can have, see [10–12].

The dynamics along the discontinuity line $x = 0$ is defined according to the Filippov's conventions, see [8]. If $F_1^-(0, y)F_1^+(0, y) > 0$, then both vector fields have the same direction, and the point $(0, y)$ is called a crossing point, and the cross region is defined as follows:

$$\Sigma^c = \{(0, y) \in \Sigma \mid F_1^-(0, y)F_1^+(0, y) > 0\}.$$