Hindawi
Complexity
Volume 2021, Article ID 6227955, 22 pages
https://doi.org/10.1155/2021/6227955

Research Article

Configuration of Zeros of Isochronous Vector Fields of Degree 5

Julio Cesar Avila,[1] **Montserrat Corbera**,[2] **and Martín Eduardo Frías–Armenta**[3]

[1]*Tecnologico de Monterrey, Campus Sonora Norte, Blvd. Enrique Mazón López 965, C. P. 83000, Hermosillo, Sonora, Mexico*
[2]*Departament D'Enginyeries, Universitat de Vic-Universitat Central de Catalunya (UVic-UCC), C. de La Laura 13, 08500 Vic, Barcelona, Spain*
[3]*Departamento de Matemáticas, Universidad de Sonora, Hermosillo, Mexico*

Correspondence should be addressed to Julio Cesar Avila; jcavila@tec.mx

Received 17 July 2021; Accepted 31 August 2021; Published 30 November 2021

Academic Editor: Baltazar Aguirre Hernandez

In this paper, we give the algebraic conditions that a configuration of 5 points in the plane must satisfy in order to be the configuration of zeros of a polynomial isochronous vector field. We use the obtained results to analyze configurations having some of its zeros satisfying some particular geometric conditions.

1. Introduction

We start defining an isochronous vector field, and we express its general associated 1-form, with its respective residues.

An isochronous vector field X is as a complex polynomial vector field on $\mathbb{C}$ whose zeros are all isochronous centers. A center is isochronous if the periods of the trajectories surrounding it are constant.

Let X be a complex polynomial vector field on $\mathbb{C}$ of degree $n \geq 1$, nonidentically zero, as follows:

$$X = \left(b_n z^n + b_{n-1} z^{n-1} + \cdots + b_1 z + b_0\right) \frac{\partial}{\partial z} = \frac{1}{\lambda} (z - p_1) \cdots (z - p_n) \frac{\partial}{\partial z}, \tag{1}$$

where the coefficients can be calculated by Vieta's formulas, in particular $\lambda = 1/b_n$. An isochronous vector field X is characterized by their associated 1-form:

$$\eta = \frac{dz}{b_n z^n + b_{n-1} z^{n-1} + \cdots + b_1 z + b_0} = \frac{\lambda dz}{(z - p_1) \cdots (z - p_n)}, \tag{2}$$

which has a unique zero at infinity of multiplicity $n - 2$ and simple poles with nonzero pure imaginary residues. For $n \geq 2$, the residue of η at p_j is

$$r_j = \frac{\lambda}{\left(p_j - p_1\right) \cdots \widehat{\left(p_j - p_j\right)} \cdots \left(p_j - p_n\right)}, \tag{3}$$

where the hat $\widehat{(p_j - p_j)}$ means that the factor $(p_j - p_j)$ is omitted (see [1]).

The following well-known result characterizes the polynomial isochronous vector fields.

Theorem 1 (see [1, 2]). *Let X be a complex polynomial vector field on $\mathbb{C}$ of degree $n \geq 2$ defined as in (1); then, the following statements are equivalent:*