Electronic Journal of Qualitative Theory of Differential Equations
2021, No. **35**, 1–89; https://doi.org/10.14232/ejqtde.2021.1.35 www.math.u-szeged.hu/ejqtde/

Structurally unstable quadratic vector fields of codimension two: families possessing a finite saddle-node and an infinite saddle-node

Joan C. Artés[1], **Marcos C. Mota**[✉2] and **Alex C. Rezende**[3]

[1]Departament de Matemàtiques, Universitat Autònoma de Barcelona,
08193, Bellaterra, Barcelona, Spain
[2]Departamento de Matemática, Instituto de Ciências Matemáticas e de Computação, Universidade de São Paulo, Avenida Trabalhador São-carlense, 400, Centro, 13.566-590, São Carlos, SP, Brazil
[3]Departamento de Matemática, Universidade Federal de São Carlos,
Rodovia Washington Luís, Km 235, Jardim Guanabara, 13.565-905, São Carlos, São Paulo, Brazil

Received 9 December 2020, appeared 15 April 2021
Communicated by Gabriele Villari

Abstract. In 1998, Artés, Kooij and Llibre proved that there exist 44 structurally stable topologically distinct phase portraits modulo limit cycles, and in 2018 Artés, Llibre and Rezende showed the existence of at least 204 (at most 211) structurally unstable topologically distinct codimension-one phase portraits, modulo limit cycles. Artés, Oliveira and Rezende (2020) started the study of the codimension-two systems by the set (AA), of all quadratic systems possessing either a triple saddle, or a triple node, or a cusp point, or two saddle-nodes. They got 34 topologically distinct phase portraits modulo limit cycles. Here we consider the sets (AB) and (AC). The set (AB) contains all quadratic systems possessing a finite saddle-node and an infinite saddle-node obtained by the coalescence of an infinite saddle with an infinite node. The set (AC) describes all quadratic systems possessing a finite saddle-node and an infinite saddle-node, obtained by the coalescence of a finite saddle (respectively, finite node) with an infinite node (respectively, infinite saddle). We obtain all the potential topological phase portraits of these sets and we prove their realization. From the set (AB) we got 71 topologically distinct phase portraits modulo limit cycles and from the set (AC) we got 40 ones.

Keywords: quadratic differential system, structural stability, codimension two, phase portrait, saddle-node.

2020 Mathematics Subject Classification: Primary: 34A34, 34C23, 34C40. Secondary: 58-02.

1 Introduction and statement of the main results

Mathematicians are fascinated in closing problems. Having a question solved or even sign with a "q.e.d" a question asked in the past is a pleasure which is directly proportional to the time elapsed between the formulation of the question and the moment of the answer.

✉Corresponding author. Email: coutinhomotam@gmail.com