Global Analysis of Riccati Quadratic Differential Systems

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In this paper, we study the family of quadratic Riccati differential systems. Our goal is to obtain the complete topological classification of this family on the Poincaré disk compactification of the plane. The family was partially studied before but never from a truly global viewpoint. Our approach is global and we use geometry to achieve our goal. The geometric analysis we perform is via the presence of two invariant parallel straight lines in any generic Riccati system. We obtain a total of 119 topologically distinct phase portraits for this family. Furthermore, we give the complete bifurcation diagram in the 12-dimensional space of parameters of this family in terms of invariant polynomials, meaning that it is independent of the normal forms in which the systems may be presented. This bifurcation diagram provides an algorithm to decide for any given quadratic system in any form it may be presented, whether it is a Riccati system or not, and in case it is to provide its phase portrait.

Keywords: Quadratic vector fields; bifurcation; topological equivalence; Riccati system; Poincaré compactification; affine invariant polynomial; configuration of invariant lines.

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