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Geometric Configurations of Singularities of Planar Polynomial Differential Systems

A Global Classification in the Quadratic Case

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We dedicate this book to the memory of the mathematician

Constantin Sibirschi (1928–1990)

on the occasion of the 90*th anniversary of his birth. Without the theory of algebraic invariants of polynomial differential equations, founded by Sibirschi, this book could not have been written.*

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Preface

In this book we consider planar polynomial differential systems, i.e. systems of the form

$$\frac{dx}{dt} = p(x, y), \qquad \frac{dy}{dt} = q(x, y)$$

where $p(x, y), q(x, y)$ are polynomials in x, y with real coefficients. To each such system there corresponds a point in $\mathbb{R}^N$ determined by its $N = (n + 1)(n + 2)$ coefficients, where n is the degree of the system, i.e. $n = \max(\deg(p), \deg(q))$. A system of degree 2 is called *quadratic*.

The study of these differential systems always begins with the study of their singularities, finite or infinite, followed by the study of separatrix connections and of limit cycles. Also in some particular cases, the study of first integrals, algebraic invariant curves and period function is of great interest.

Our main goal in this book is to classify in a geometrical way the global schemes of singularities, finite and infinite, of quadratic differential systems and to obtain their bifurcation diagram in the 12-dimensional space $\mathbb{R}^{12}$. This global classification and its bifurcation diagram is completely algebraic, and we provide the algorithm that computes, for every family of quadratic systems, the global bifurcation diagram of its corresponding schemes of singularities. The study of singularities is the first step in the topological classification of the phase portraits of these differential systems and their bifurcation diagram. The geometrical equivalence relation between singularities considered here, is deeper than the topological one, including features of an algebraic-geometric meaning that play a significant role in studying bifurcations of the systems.

This was a long-term project. Our work began seven or even eight years ago. Every year we met in the spring in Barcelona, then in the fall in Montreal, in order to work on the project. During the past three years, two of us met in late summer in Chişinău, Moldova. We were happy to have the opportunity to work together and in the acknowledgements we mention the institutions and grants that supported us.

Over the years, we published partial results such as the study of infinite singularities, then of quadratic systems with total multiplicity of finite singularities less than or equal to one, or with total multiplicity of finite singularities equal to two or three. From the class of quadratic differential systems with total multiplicity of finite singularities equal to four, those with total number of distinct finite singularities less than or equal to three, were also published. On one of these last published articles, we worked together with Alex C. Rezende, and we thank him for his contribution to our project.

The original results appearing in the book in Chapters 7, 12 and in Section 11.4 of Chapter 11 have never been published before and so they appear here for the first time. Section 11.4 contains the most generic and most difficult cases. This classification yielded 1765 distinct geometrical configurations of singularities, finite or infinite, plus at most 8 other such configurations (sharing the same finite part) that we conjecture are not realizable.

We give in the final chapter of the book some concluding comments with a view towards the future.

We are thankful to the editors and referees for the improvements they suggested and their advice was followed by us.

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