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Uniqueness of the limit cycles for complex differential equations with two monomials

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ABSTRACT

We prove that any complex differential equation with two monomials of the form $\dot{z} = az^k\bar{z}^l + bz^m\bar{z}^n$, with k, l, m, n non-negative integers and $a, b \in \mathbb{C}$, has one limit cycle at most. Moreover, we characterise when such a limit cycle exists and prove that then it is hyperbolic. For an arbitrary equation of the above form, we also solve the centre-focus problem and examine the number, position, and type of its critical points. In particular, we prove a Berlinskiĭ-type result regarding the geometrical distribution of the critical points stabilities.

1. Introduction

In this work, we will prove that any complex polynomial differential equation inside the family

$$\dot{z} = az^k\bar{z}^l + bz^m\bar{z}^n, \ z \in \mathbb{C}, \tag{1.1}$$

with $k, l, m, n \in \mathbb{Z}^+ \cup \{0\}, k + l < m + n$, and $a, b \in \mathbb{C}\backslash\{0\}$ has one limit cycle at most. Recall that a limit cycle γ is a periodic orbit such that, in at least one of the connected components of $\mathbb{R}^2 \setminus \gamma$, has initial conditions (as close to γ as desired) that do not belong to a periodic orbit.

Note that easier cases $k + l = m + n$ or $ab = 0$ need not be considered because they give rise to particular planar homogeneous vector fields and the global phase portraits of the general homogeneous polynomial

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