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Research paper

Dynamics in a time-discrete food-chain model with strong pressure on preys[☆]

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ABSTRACT

Discrete-time dynamics, mainly arising in boreal and temperate ecosystems for species with non-overlapping generations, have been largely studied to understand the dynamical outcomes due to changes in relevant ecological parameters. The local and global dynamical behaviour of many of these models is difficult to investigate analytically in the parameter space and, typically, numerical approaches are employed when the dimension of the phase space is large. In this article we provide topological and dynamical results for a map modelling a discrete-time, three-species food chain with two predator species interacting on the same prey. The domain where dynamics live is characterised, as well as the so-called escaping regions, which involve species extinctions. We also provide a full description of the local stability of equilibria within a volume of the parameter space given by the prey's growth rate and the predation rates. We have found that the increase of the pressure of predators on the prey results in chaos via a supercritical Neimark-Sacker bifurcation. Then, period-doubling bifurcations of invariant curves take place. Interestingly, an increasing predation directly on preys can shift the extinction of top predators to their survival, allowing an unstable persistence of the three species by means of periodic and chaotic attractors.

1. Introduction

Ecological systems display complex dynamical patterns both in space and time [1]. Although early work already pointed towards complex population fluctuations as an expected outcome of the nonlinear nature of species' interactions [2,3], the first evidences of chaos in species dynamics was not characterised until the late 1980's and 1990's [4,5]. Since pioneering works on one-dimensional discrete models [6–9] and on time-continuous ecological models, e.g., with the so-called spiral chaos [10,11] (already pointed out by Rössler in 1976 [12]), the field of ecological chaos experienced a strong debate and

[☆] Dedicated to Mitchell Jay Feigenbaum, *in memoriam*.

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