LOWER BOUNDS OF THE TOPOLOGICAL ENTROPY OF MAPS OF Y[1]

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Abstract. We give the best lower bound of the topological entropy of a continuous map f of the space $\mathbf{Y} = \{ z \in \mathbf{C} \mid z^3 \in [0,1] \}$ into itself, with $f(0) = 0$, as a function of its set of periods.

Let $\mathcal{Y}$ be the family of continuous maps of the space $\mathbf{Y} = \{ z \in \mathbf{C} \mid z^3 \in [0,1] \}$ into itself with 0 as a fixed point. A characterization of the set of periods of periodic orbits of $f \in \mathcal{Y}$ based upon the knowledge of the behaviour of certain periodic orbits was given in [ALM].

Having the characterization of the behaviour of those periodic orbits, we can apply the standard techniques of [BGMY] to calculate the best lower bounds of topological entropy for $f \in \mathcal{Y}$, depending on the set of periods of f. Thus our work goes in the fifth of the six directions suggested in [ALM].

As usual $x \in \mathbf{Y}$ is a *periodic point* for $f \in \mathcal{Y}$ if there is some $n \in \mathbf{N}$ such that $f^n(x) = x$. Then the set $\{ x, f(x), \dots, f^{n-1}(x) \}$ is a *periodic orbit* of f, and its *period* is the smallest $m \in \mathbf{N}$ such that $f^m(x) = x$. We denote by $Per(f)$ the set of periods of all periodic orbits of f.

We define several orderings of some subsets of $\mathbf{N}$. In the whole paper the symbol $\equiv$ will denote congruence mod 3.

- The *Šarkovskiĭ ordering* of $\mathbf{N}$ is:

$$3,\ 5,\ 7,\ 9,\ \dots,\ 2\cdot3,\ 2\cdot5,\ 2\cdot7,\ 2\cdot9,\ \dots,\ 2^2\cdot3,\ 2^2\cdot5,\ 2^2\cdot7,\ 2^2\cdot9,\ \dots,\ \dots,\ 2^3,\ 2^2,\ 2,\ 1.$$

If k appears to the right of n in the above ordering, we shall write $k >_s n$. If $k = 2^p \cdot k'$ and $n = 2^q \cdot n'$, where k' and n' are odd, then we have $k >_s n$ if and only if one of the following cases occurs:

(i) $k' > 1$, $n' > 1$, $p > q$.
(ii) $k' > 1$, $n' > 1$, $p = q$, $k' > n'$.
(iii) $k' = 1$, $n' > 1$.
(iv) $k' = 1$, $n' = 1$, $p < q$.

[1]This is a summary of [AM]