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Kneading theory and rotation intervals for a class of circle maps of degree one†

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Abstract. We give a kneading theory for the class of continuous maps of the circle of degree one with a single maximum and a single minimum. For a map of this class we characterise the set of itineraries depending on the rotation interval. From this result we obtain lower and upper bounds of the topological entropy and of the number of periodic orbits of each period. These lower bounds appear to be valid for a general continuous map of the circle of degree one.

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1. Introduction and statement of the results

In this paper we study the class $\mathcal{A}$ of maps which are liftings of a circle map of degree one with a single maximum and a single minimum in $[0,1]$ (a precise definition of class $\mathcal{A}$ will be given later). The study of these maps arises naturally in different contexts in dynamical systems. For instance, a three-parameter family of maps from class $\mathcal{A}$ has been used by Levi (see [15]) to study Van der Pol equation (see also [1, 5]). Also, they are relevant in the description of the transition to chaos for area-contracting maps of an annulus.

One of the main tools to study the dynamics of circle maps of degree one is the rotation interval. It is a generalisation of the rotation number introduced by Poincaré to study homeomorphisms of the circle. Roughly speaking the rotation interval is the set of all average angular speeds of points under iteration of the map and it gives a lot of information about the periodic orbits of the map under consideration (see [8, 18, 22]). Another important notion is the topological entropy. It characterises the complexity of the maps. Intuitively it measures the exponential growth rate of the number of periodic orbits as we increase their periods [19]. For a piecewise monotone map it is also the logarithm of the growth number which is the exponential growth rate of the number of pieces of monotonicity of the iterates of the map [19]. The fact that both, the rotation interval and the topological entropy have something to do with

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