

## PERIODS AND ENTROPY FOR LORENZ-LIKE MAPS

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### 1. Notation, definitions and statement of results.

In the paper we shall use the following notations. When we write  $p/q$  we mean that  $p, q \in \mathbf{Z}$  and  $q > 0$ . If we write  $k > 0$  or  $k \geq 0$ , we mean that additionally  $k \in \mathbf{Z}$ . The greatest common divisor of  $p$  and  $q$  will be denoted by  $(p, q)$ . If  $A$  is a subset of  $\mathbf{N} = \{1, 2, 3, \dots\}$  then  $kA$  will denote the set  $\{ka : a \in A\}$ . We shall denote by  $E(\cdot)$  the integer part function.

We denote by  $e : \mathbf{R} \rightarrow \mathbf{S}^1 = \{z \in \mathbf{C} : |z| = 1\}$  the natural projection  $e(x) = \exp(2\pi ix)$  (here  $i = \sqrt{-1}$ ). A map  $F : \mathbf{R} \rightarrow \mathbf{R}$  is called a *lifting* of a map  $f : \mathbf{S}^1 \rightarrow \mathbf{S}^1$  if  $e \circ F = f \circ e$  and there is  $k \in \mathbf{Z}$  such that  $F(x+1) = F(x) + k$  for all  $x \in \mathbf{R}$ . This  $k$  is called the *degree* of  $F$ . Note that since we do not say anything about continuity here, every  $f$  has liftings of all degrees.

A map  $F : \mathbf{R} \rightarrow \mathbf{R}$  will be called *old* if  $F(x+1) = F(x) + 1$  for all  $x \in \mathbf{R}$  (here we follow the terminology of [M3]; old stands for «degree one lifting» with the order of letters changed for mnemonic reasons). It is easy to see that if  $F$  is an old map then  $F(x+k) = F(x) + k$  for all  $x \in \mathbf{R}$  and  $k \in \mathbf{Z}$ , and that the iterates of an old map are old maps.

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