

## VII C.E.D.Y.A.

TWIST PERIODIC ORBITS AND TOPOLOGICAL ENTROPY FOR CONTINUOUS  
MAPS OF THE CIRCLE OF DEGREE ONE WHICH HAVE A FIXED POINT<sup>(\*)</sup>

by

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Let  $S^1$  be the circle. We denote by  $C_1(S^1)$  the set of all continuous maps from  $S^1$  to itself of degree one. For  $x \in S^1$ , we say that  $x$  is periodic if there exists a positive integer  $n$  such that  $f^n(x) = x$ . The period of  $x$  is the smallest integer satisfying this relation. Let  $P(f)$  be the set of periods of  $f$ . If  $x \in S^1$  is a periodic point of period  $n$ , then the orbit of  $x$  is the set  $\{f^k(x) : k=1, 2, \dots, n\}$ . We refer to such an orbit as a periodic orbit of period  $n$ .

Let  $f \in C_1(S^1)$ ,  $F$  its lifting to the covering space  $\mathbb{R}$  and  $e(X) = \exp(2\pi iX)$  the natural projection of  $\mathbb{R} \rightarrow S^1$ . We note that  $F$  is not defined uniquely; nevertheless, if  $F$  and  $F'$  are two liftings of  $f$  then  $F = F' + m$  with  $m \in \mathbb{Z}$ . Since  $\deg(f) = 1$  we have  $F(X + 1) = F(X) + 1$  for all  $X \in \mathbb{R}$ . If  $x$  is a periodic point of  $f$  of period  $n$  and  $e(X) = x$ , then  $F^n(X) = X + k$  where  $k \in \mathbb{Z}$ . We shall call  $k/n$  the *rotation number* (or  $F$ -rotation number, if necessary) of  $x$  and we denote it by  $\rho(x)$  or  $\rho_F(x)$ . We denote by  $L(f)$  or  $L_F(f)$  the set of all rotation numbers of  $f$ .

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