

European Conference on **ITERATION THEORY** (ECIT 87)

Caldes de Malavella, Spain 20-26 September 1987

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INTRODUCTION

This volume contains the Proceedings of the *European Conference on Iteration Theory* (ECIT 87) that was held at Caldes de Malavella, a small village with old roman baths, near Barcelona (Spain), from September 20 to 26, 1987.

It was the sixth in a series of international meetings on iteration theory, following Toulouse (1973), Schloss Retzhof near Graz (1977), Amöneburg near Marburg (1980), again Toulouse (1982) and Lochau (Voralberg, Austria) in 1984. The next meeting (ECIT 89) is scheduled for September 10 to 16, 1989, in Batschuns (Voralberg, Austria). We hope that the series will continue for a long time.

Indeed iteration theory is a wide field which includes topics on discrete dynamical systems and several problems of functional equations. Also most of the methods in numerical analysis rely on iteration. Iteration theory in a large sense studies all the aspects of the following problem: take a function (invertible or not, smooth or not) and an initial point. Then look for the image of the point under the function and for the successive images. That is, one iterates the algorithm. Several problems come to the mind. Which is the structure of the set of points obtained? Are there fixed points? Are there periodic points which return to one of the previous points in the sequence? What about the stability of those points? What about the full set of sequences that can be obtained? Is the structure of this set robust when the function is slightly changed? Is it possible to conjugate the function by means of a change of variable to another one that is simpler or easier to study?

The behaviour of many phenomena in the physical, natural or social sciences can be described by means of functions which tell us which will be the state of the system at some epoch in terms of the state at a previous time. Then to understand, make predictions and be able to control those phenomena, it is interesting to learn about iteration theory. As mentioned before iteration theory also plays an important role in numerical analysis. There one derives an algorithm to solve some equation (either numerical or functional). It is important to know not only that the algorithm is convergent to some solution but also the speed of convergence. This is related to the stability properties of a fixed point under the iteration function. Also it is important to know the set of initial points converging to a given solution, that is, the basin of attraction of the fixed point. The structure of this basin can be extremely complicated with infinitely many connected components having boundaries of fractal dimension. Despite the fact that several communications in this meeting contain material that can be applied to numerical analysis problems, those applications cannot be found here.

Systems evolving in a continuous way can be discretized if one looks at them at given intervals of time or when the state passes through a prescribed manifold (this is the well-known method of the surfaces of section due to Poincaré). So many problems in differential equations can be studied using the results of iteration theory.

The converse problem can also be posed. Is it possible, given an invertible iteration function, to embed it in a flow? Or, if the function is not invertible, to embed it on a semiflow?

These Proceedings contain several one-hour lectures which are mainly of survey type on the subject, most of the communications or short communications presented at ECIT 87, and some abstracts. The topics are extremely diverse including, among others, one-dimensional maps, topological entropy, two-dimensional diffeomorphisms and endomorphisms, annulus maps, renormalization, bifurcations, resonances, functional equations, functional analysis, iteration of formal power series, roots of automorphisms, polynomial vector fields, homoclinic orbits and transversality. We refer the readers to the table of contents for the complete list of topics.

The organisers are grateful to the program SCIENCE of the EEC. A twinning grant to their home institutions made possible that several persons of these institutions attend the meeting. This constituted more than fifty per cent of the full audience. Other participants were supported by Spanish grants to the local organisers (C.A., J.Ll., C.S.).

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Barcelona, Marburg and Toulouse, May 1989

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