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KNEADING THEORY OF LORENZ MAPS(*)

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ABSTRACT: In this paper we describe the use of the kneading theory in the study of the dynamics of one-dimensional maps, with special emphasis on the periodic behaviour and the topological entropy.

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In the study of the geometrical model of the Lorenz attractor, a class of one-dimensional maps plays an important role. We refer to such maps as the Lorenz maps (see [GH] , [Sp] and [T]) although they are different from the one-dimensional maps presented by Lorenz (see [Lo]). We shall introduce the kneading theory to study the dynamics of Lorenz maps.

1. Lorenz maps.

Let $I = [-1,1]$. We shall say that a map $f : I \longrightarrow I$ is Lorenz if

L1) f has a single discontinuity at 0, $\lim_{x\to 0^+} f(x) = -1$ and $\lim_{x\to 0^-} f(x) = 1$,

L2) f is odd on $I\setminus\{0\}$ (i.e. $f(-x) = -f(x)$ for all $x \in I\setminus\{0\}$),

L3) $f(-1) < 0$ and $f(0) = 1$,

L4) f is once continuously differentiable on $I\setminus\{0\}$, and $f'(x) > 1$ for all $x \in I\setminus\{0\}$.

Note that every Lorenz map is strictly monotone on the intervals $[-1,0)$ and $(0,1]$. Many of our results would also be true for maps satisfying L1), L2), L3) and

L5) f is strictly increasing on $[-1,0)$ and $(0,1]$,

instead of L4); but the ideas of some proofs seem more transparent for Lorenz maps. It is also easy to see that the particular choice of the interval $[-1,1]$ and fixing the discontinuity at $x = 0$ involves no loss of generality.

2. Itineraries.

Let f be a Lorenz map. For a point $x \in I$ we define its itinerary (an infinite sequence of symbols I_n),

$$\underline{I}(x) = \underline{I}_f(x) = I_0 I_1 I_2 \dots,$$

(*) This is a summary of [AL] .